RESEARCH IN ROBUST CONTROL FOR HYPERSONIC AIRCRAFT

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Principal Investigator: A.J. Calise
Research Assistant: H. Buschek
NASA Grant Monitors: J.D. McMinn
                      I. M. Gregory

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

Summary

The research during the third reporting period has focused on fixed order robust control design for hypersonic vehicles. A new technique has been developed to synthesize fixed order H_∞ controllers. A controller canonical form is imposed on the compensator structure and a homotopy algorithm is employed to perform the controller design. Various reduced order controllers are designed for a simplified version of the hypersonic vehicle model used in our previous studies to demonstrate the capabilities of the code. However, further work is needed to investigate the issue of numerical ill-conditioning for large order systems and to make the numerical approach more reliable.

1 Introduction

The objective of this research is to address the issues associated with the design of robust integrated flight control systems for future hypersonic vehicles with airbreathing propulsion systems. It is anticipated that such vehicles will exhibit significant interactions between rigid body (airframe) dynamics, structural dynamics and engine dynamics. The uncertainty in the initial dynamic models developed for these vehicles will also be high. The main reason that highly interactive uncertain dynamics are to be expected is that scramjet engines will be the primary source of propulsion at hypervelocity speeds. Wind-tunnel testing as a result will be limited, and it will be necessary to gain experience in actual flight testing of such vehicles. This means that initial flight control system design efforts will rely more heavily on theoretical and computer based models, than has been the case for subsonic and supersonic aircraft. Also, propulsion system sensitivity to angle of attack variations and structural vibrations will lead to highly interactive dynamics.

In this study, the current major research issues from a flight control viewpoint are: (1) the development of models that are representative of the interactive dynamics that can occur in such vehicles, (2) the development of representative uncertainty models for these dynamics and (3) the development of practical approaches to designing multivariable flight control systems that guarantee adequate performance in the presence of uncertainty. The research done during the third reporting period has been focusing on item (3).

The hypersonic vehicle model used in this study [1] neglects both the effects of angle of attack variations on propulsion system performance and of elastic body bending. In the framework of robust control design these effects are treated as uncertainties. This study has resulted in uncertainty models capturing the individual characteristics of these phenomena. The models were developed and incorporated into a control design structure which evolved from an earlier study described in Ref. [2]. A variety of robust controllers were designed utilizing H_∞ and μ-synthesis techniques and the sensitivity of achievable robust performance to the introduced uncertainty levels were investigated. A thorough description as well as a comprehensive discussion of the results is given in Ref. [3]. While demonstrating the feasibility of modern control techniques for hypersonic control system design, the results in Ref. [3] also indicate that a significant drawback of these methods will be the large order of the resulting controller. When implemented, large order controllers can create time delays which may be undesirable. Furthermore, it was shown that model order reduction techniques to reduce controller order do not guarantee robust performance.

These results demonstrate the need for fixed order controller design. A technique to synthesize H_∞ controllers with a constraint on controller order is developed in Ref. [4]

which was submitted to the 1994 AIAA Guidance, Navigation, and Control Conference in Scottsdale, AZ, (see Appendix). Therefore, this report provides only a brief summary of the research performed and focuses on a discussion of future research.

2 Fixed Order H_∞ Controller Design

The objective of fixed order design is to constrain the order of the controller a priori in the design process and to synthesize a robust controller subject to this constraint. In our case, a controller canonical form was imposed on the compensator structure. This leads to a closed loop formulation of the problem with a minimal number of parameters and converts it to a static gain output feedback problem.

In Ref. [5] a conjugate gradient method was used for the numerical solution of this problem. Besides the issue of slow convergence near the optimum, this method requires an initial starting guess which is sometimes hard to find. Thus, in our approach a homotopy algorithm is used to simplify this procedure. A homotopy starts with a known or easily calculated solution to a simple problem and deforms it gradually to the solution of the desired, but more complex problem while pursuing a so-called homotopy path. Here, the solution to the simple problem is a reduced order H_2 controller designed for low control authority. Using a two-step homotopy procedure, the desired fixed order H_∞ controller for the original formulation is obtained. A detailed derivation of the problem formulation as well as an introduction into the homotopy algorithm used is given in Ref. [4].

The developed algorithm to synthesize fixed order H_∞ controllers was applied to the Winged-Cone Configuration used in our earlier studies [1]. However, in order to perform initial calculations a simplified version of the "full-scale" model developed in Ref. [3] was used. Hence, the achieved results demonstrate the capabilities of the developed algorithm but do not necessarily represent realistic responses. Our future work will eliminate this shortcoming. Also, the current version of the algorithm is sensitive to numerical ill-conditioning for large order plants such as the one used in this study. Currently, research is under way to robustify the algorithm and to make the numerical approach more dependable.

The uncertainty models developed in Ref. [3] representing propulsion system perturbations, flexible body bending, and uncertainty in control effectiveness are not included in these early stages of fixed order controller design. Since an H_∞ design does not take into account the structure in the uncertainty, such a design would be too conservative in order to achieve robust performance. This is illustrated in Fig. 1 where the full order μ design

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from Ref. [3] is compared to an equivalent full order H_∞ design. The uncertainty levels are 25% uncertainty in $c_{M\alpha}$ due to propulsive effects, uncertainty in the angle of attack caused by elastic deformation with the first body bending mode at 18.5 rad/sec, and 10% uncertainty in control effectiveness in both control channels. Clearly, the H_∞ design does not achieve robust performance while the μ design does. This demonstrates the necessity to develop a fixed order robust control design technique that considers the structure in the uncertainty.
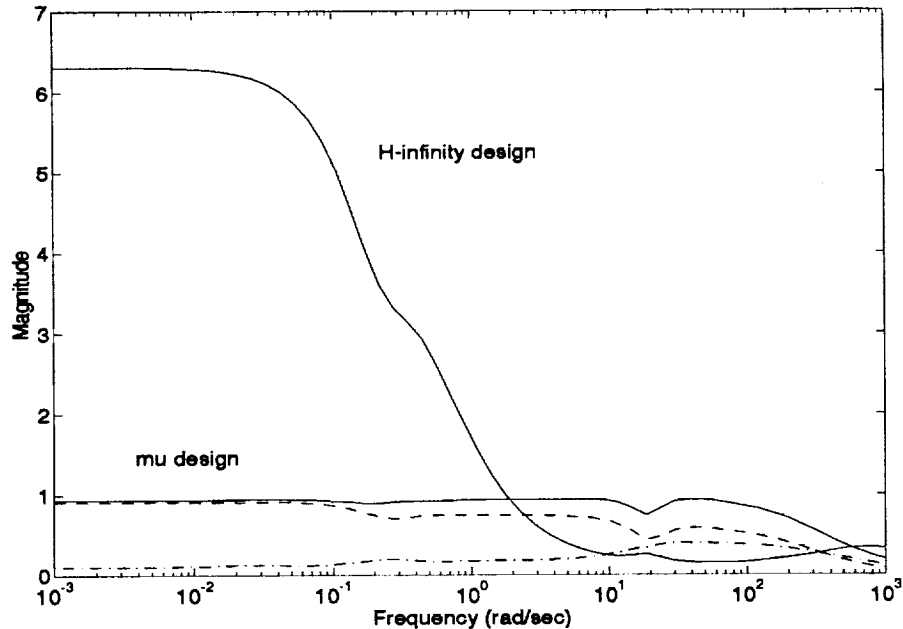

Figure 1: Robust performance bounds: H_∞ design vs. μ design. (Also given: NP bounds (dash) and RS bounds (dash-dot) for μ design)

3 Future Research

The results achieved with the methodology described in the previous section are quite encouraging and the approach has the potential for further development. The need for fixed order robust control design for structured uncertainty is obvious, but no such technique presently exists. Our future work will focus on the development of such a methodology.

One possibility to pursue this task would be to replace the full order H_∞ design step in the μ-synthesis procedure with the fixed order technique. Since H_∞ controller design is a subproblem when designing for robust performance with structured uncertainty, the fixed order technique has the potential to constrain the order of the controller which is

normally subject to significant increases in the μ-synthesis procedure. This "brute force" technique would adopt the D-K iteration procedure including the rather awkward step of curve fitting to find a rational transfer function representation of the D-scales .

Currently, research is under way to circumvent this curve fitting phase in order to generate a completely automated design procedure for robust controller synthesis. In Ref. [6] the D-scales are selected to be constant instead of frequency dependent. This eliminates the need for curve fitting, and, moreover, allows performing a simultaneous optimization of D-scales and controller making a D-K iteration superfluous. Naturally, constant D-scales are more conservative than frequency dependent ones. Dynamic D-scales of a prespecified order are used by the same authors in Ref. [7]. A state space representation of the dynamic D-scales is formulated and the optimal D-scales are computed. An attempt to simultaneously solve for optimal dynamic D-scales and an optimal fixed order controller using the formulation in Refs. [6] and [7] was not undertaken and would have lead to a large system of coupled equations. The numerical solution of such a system is rather difficult or may even be impossible. The canonical representation with its minimal number of parameters has the potential to greatly simplify the formulation of this problem. Furthermore, the optimization procedure used in Refs. [6] and [7] would naturally fit into the general homotopy framework used in the H_∞ design. This issue will be investigated further.

A different approach to bypass the curve fitting step is pursued in Refs. [8] and [9] where the authors use K_m-synthesis for robust controller design. Employing a bilinear sector transform, the D-scales can be replaced by generalized Popov multiplier matrices M of a prespecified order. Similar to the D-K iteration in μ-synthesis , a M-F iteration is performed where alternately full order H_∞ controllers (F) and optimal multipliers (M) are computed. Embedded in this procedure is a fixed order multiplier optimization that replaces the curve fitting. The entire optimization is not simultaneous as in Ref. [6] and does not yield reduced order compensators, but it allows for controller synthesis using dynamic multipliers. Another major advantage of this technique is that both complex and real parameter uncertainty can be treated. The capacity to model variations in real parameters is considered a primary contribution that reduces conservatism in the design even further. Improvements in this area are currently the subject of expansive research activities [10] - [13]. In K_m-synthesis real uncertainty representation is accomplished by the utilization of the generalized Popov multiplier which allows for the introduction of phase information into the design. This technique is comparable to the approaches taken in Refs. [12] and [13]. It is not yet clear how the generalized Popov multiplier formulation can be included in the desired homotopy/fixed order design framework. However, it offers an interesting perspective especially under the aspect of including real parameter uncertainty.

An in depth application to the hypersonic vehicle model used in our previous work will be studied to evalute the developed techniques, and to investigate the critical modeling and design issues related to fixed order robust control design for this class of vehicles.

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Appendix

Fixed Order Robust Control Design for Hypersonic Vehicles

Harald Buschek* and Anthony J. Calise**

Georgia Institute of Technology

Aerospace Engineering

Atlanta, GA 30332

Abstract

Control system design for hypersonic vehicles using modern control theory techniques generally leads to high order controllers. The time delays which are created when these controllers are implemented may not be acceptable. A technique to constrain controller dimension apriori in the design process employing a controller canonical realization and a homotopy algorithm is used to design fixed order H_∞ controllers. Various reduced order controllers are designed for a hypersonic vehicle model accelerating through Mach 8. A comparison with a full order design is conducted and the results indicate that the developed fixed order design technique is suitable for hypersonic applications.

*Graduate Research Assistant, Student Member AIAA.
Phone (404) 894-3022, e-mail buschek@isc11.ae.gatech.edu
**Professor, Fellow, AIAA.
Phone (404) 894-7145, Fax (404) 894-2760, e-mail anthony.calise@aerospace.gatech.edu

1   Introduction

At the dawn of the next millenium, hypersonic flight is one of the most demanding frontiers challenged by aerospace scientists. Designing a vehicle that has horizontal take-off and landing capability, is able to perform hypersonic atmospheric flight and reaches a typical low earth orbit involves considering a variety of different effects ands requirements. A crucial component of such an aerospace vehicle is the control system which stabilizes the vehicle and ensures precise tracking of pilot commanded inputs. This task will be further complicated by a strong interaction between aerodynamics, structure, and the propulsion system.

It has been shown in Ref. [1] that modern control theory using H_∞ and μ-synthesis techniques is well suited for addressing multiple uncertainty sources in hypersonic flight control design. However, a significant disadvantage of these techniques is that the resulting controller is of the same order as the generalized plant. Frequency dependent weights have to be included in the design framework in order to achieve the desired performance and robustness characteristics. Thus, the order of the generalized plant is increased resulting in high order controllers. When implemented, large order controllers can create time delays which may be undesirable. One solution to this problem is to use model order reduction on the controller realization. This technique, though, does not consider the properties of the closed loop system when reducing the order of the controller and therefore robustness properties are not guaranteed. Another approach to this problem is to constrain the order of the controller apriori in the design process.

In this paper, a controller canonical form is imposed on the compensator structure to constrain the controller dimension. Necessary conditions for an optimal H_∞ controller are derived using a differential game formulation. A homotopy algorithm is used to continuously deform the solution of an H_2 problem formulation which serves as starting point to

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the solution of the desired problem formulation, i.e. an optimal fixed order H_∞ controller. This technique is applied on a hypersonic vehicle accelerating through Mach 8. Several fixed order H_∞ controllers are obtained and their performance is compared to a full order design.

2 Fixed Order Compensators for the H_∞ Problem

For a standard control problem, the generalized plant is given by

$$\dot{x} = Ax + B_1 w + B_2 u \tag{1}$$

$$z = C_1 x + D_{12} u \tag{2}$$

$$y = C_2 x + D_{21} w + D_{22} u \tag{3}$$

where $x \in \Re^n$ is the state vector, $w \in \Re^{n_w}$ is the disturbance vector, $u \in \Re^{n_u}$ is the control vector, $z \in \Re^{n_z}$ is the performance vector, and $y \in \Re^{n_y}$ is the observation vector. It is assumed that

- (A, B_1, C_1) is stabilizable and detectable

- (A, B_2, C_2) is stabilizable and detectable

- D_{12} has full column rank

- D_{21} has full row rank.

A general compensator for this system is .

$$\dot{x}_c = A_c x_c + B_c y \tag{4}$$

$$u = C_c x_c \tag{5}$$

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where $x_c \in \Re^{nc}$ is the state vector of the controller the dimension of which can be specified. Fig. 1 illustrates this design framework.

The drawback with this general controller formulation is that the problem is over-parametrized. To avoid this problem of overparametrization, a canonical form description is used for the controller [2]. In controller canonical form the compensator is defined as

$$\dot{x} = P^0 x_c + N^0 u_c - N^0 y \qquad (6)$$

$$u_c = -P x_c \qquad (7)$$

$$u = -H x_c \qquad (8)$$

where $x_c \in \Re^{nc}$ and $u_c \in \Re^{ny}$. P and H are free-parameter matrices, and P^0 and N^0 are fixed matrices of zeros and ones determined by the choice of controllability indices ν_i as follows:

$$P^0 = \text{block diag}\{P^0_1, \ldots, P^0_{ny}\} \qquad (9)$$

$$P^0_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 1 \\ 0 & \cdots\cdots & & 0 \end{bmatrix}_{\nu_i \times \nu_i} \qquad i = 1, \ldots, ny \qquad (10)$$

$$N^0 = \text{block diag}\left\{[0\ldots01]^T_{1\times\nu_i}\right\} \quad i = 1, \ldots, ny \qquad (11)$$

The controllability indices must satisfy the following condition:

$$\sum_i \nu_i = nc \qquad i = 1, \ldots, ny \qquad (12)$$

Fig. 2 shows the structure of such a controller. Similarly, a compensator in observer canonical form can be constructed. However, in this paper only the controller canonical form is employed.

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With this formulation, the compensator states can be absorbed into the generalized plant. Let

$$\bar{x} = \begin{bmatrix} x \\ x_c \end{bmatrix} \qquad \bar{u} = \begin{bmatrix} u \\ u_c \end{bmatrix}. \tag{13}$$

The augmented system is defined by:

$$\begin{aligned}
\dot{\bar{x}} &= \begin{bmatrix} A & 0 \\ -N^0 C_2 & P^0 \end{bmatrix} \bar{x} + \begin{bmatrix} B_1 \\ -N^0 D_{21} \end{bmatrix} w + \begin{bmatrix} B_2 & 0 \\ -N^0 D_{22} & N^0 \end{bmatrix} \bar{u} \\
&= \bar{A}\bar{x} + \bar{B}_1 w + \bar{B}_2 \bar{u}
\end{aligned} \tag{14}$$

$$\begin{aligned}
z &= \begin{bmatrix} C_1 & 0 \end{bmatrix} \bar{x} + \begin{bmatrix} D_{12} & 0 \end{bmatrix} \bar{u} \\
&= \bar{C}_1 \bar{x} + \bar{D}_{12} \bar{u}
\end{aligned} \tag{15}$$

$$\bar{y} = \begin{bmatrix} 0 & I \end{bmatrix} \bar{x} = \bar{C}_2 \bar{x} \tag{16}$$

$$\bar{u} = -\begin{bmatrix} H \\ P \end{bmatrix} \bar{y} = -G\bar{y} \tag{17}$$

Eqs. 14-17 define a static gain output feedback problem where the compensator is represented by a minimal number of free parameters in the design matrix, G. The augmented system is shown in Fig. 3. The closed loop system is given by

$$\begin{aligned}
\dot{\bar{x}} &= (\bar{A} - \bar{B}_2 G \bar{C}_2)\bar{x} + \bar{B}_1 w \\
&= \tilde{A}\bar{x} + \tilde{B}w
\end{aligned} \tag{18}$$

$$z = (\bar{C}_1 - \bar{D}_{12} G \bar{C}_2)\bar{x} = \tilde{C}\bar{x} \tag{19}$$

In the H_∞ problem, the objective is to minimize the ∞-norm of the transfer function from disturbance inputs w to performance outputs z

$$T_{zw} = \tilde{C}(sI - \tilde{A})^{-1}\tilde{B}. \tag{20}$$

If $z(s) = T_{zw}(s)w(s)$, then

$$\|T_{zw}\|_\infty = \sup_{w \in L_2} \frac{\|T_{zw}w\|_2}{\|w\|_2} = \sup_{\|w\|_2 \leq 1} \|T_{zw}w\|_2. \tag{21}$$

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As described in Ref. [3], the optimization problem is to find

$$\inf \{\|T_{zw}(G)\|_\infty : G \in \mathbf{G}\} := \gamma^* \qquad (22)$$

where $\mathbf{G} = \{G \in R^{(nu+ny) \times nc} : \tilde{A} \text{ is stable}\}$. A more practical design objective would be to find a G that insures $\|T_{zw}\|_\infty \leq \gamma$ for some $\gamma > \gamma^*$. Considering the performance index

$$J_\gamma(w, G) = E\left\{\int_0^\infty (z^T z - \gamma^2 w^T w)dt\right\} \qquad (23)$$

where $E\{\bar{x}(0)\} = 0$ and $E\{\bar{x}(0)\bar{x}(0)^T\} = \tilde{B}\tilde{B}^T$, the H_∞ problem can be formulated as a min-max problem

$$J_\gamma^* = \min_{G \in \mathbf{G}_\gamma} \max_{w \in L_2} \{J_\gamma(w, G)\} \qquad (24)$$

where $\mathbf{G}_\gamma = \{G \in R^{(nu+ny) \times nc} : \tilde{A} \text{ is stable and } \|T_{zw}\|_\infty < \gamma\}$. In the min-max problem, the minimizing player acts first, and then the maximizer. Therfore, the loop is closed on $\bar{u}$ and w maximizes, whatever G might be. For any $G \in \mathbf{G}_\gamma$, a unique maximizing solution exists and is given by

$$w = \gamma^{-2}\tilde{B}^T Q_\infty \bar{x} \qquad (25)$$

where Q_∞ is the positive semidefinite solution of

$$\tilde{A}^T Q_\infty + Q_\infty \tilde{A} + \tilde{C}^T \tilde{C} + \gamma^{-2} Q_\infty \tilde{B}\tilde{B}^T Q_\infty = 0 \qquad (26)$$

Using the worst case disturbance in Eq. 25, the performance index can be rewritten as

$$J_\gamma(G) = E\left\{\int_0^\infty \bar{x}^T(\tilde{C}^T \tilde{C} - \gamma^{-2} Q_\infty \tilde{B}\tilde{B}^T Q_\infty)\bar{x}dt\right\} . \qquad (27)$$

Defining a distribution of initial conditions with zero mean and variance $\tilde{B}\tilde{B}^T$, the objective of minimizing $\|T_{zw}\|_\infty$ using a fixed order controller can be formulated

$$\min_{G \in \mathbf{G}_\gamma} \left\{J_\gamma(G) = \text{tr}\{Q_\infty \tilde{B}\tilde{B}^T\}\right\} \qquad (28)$$

subject to the Riccati equation given in Eq. 26. In order to obtain the H_∞-optimal compensator, the Lagrangian is defined as

$$\mathbf{L}(Q_\infty, L, G) = \text{tr}\left\{Q_\infty \tilde{B}\tilde{B}^T + (\tilde{A}^T Q_\infty + Q_\infty \tilde{A} + \tilde{C}^T \tilde{C} + \gamma^{-2} Q_\infty \tilde{B}\tilde{B}^T Q_\infty)L\right\} \qquad (29)$$

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where L represents the Lagrangian multiplier. Matrix gradients are taken to determine the first order necessary conditions for an H_∞-optimal fixed order controller:

$$\frac{\partial \mathbf{L}}{\partial Q_\infty} = (\tilde{A} + \gamma^{-2}\tilde{B}\tilde{B}^T Q_\infty)L + L(\tilde{A} + \gamma^{-2}\tilde{B}\tilde{B}^T Q_\infty)^T + \tilde{B}\tilde{B}^T = 0 \qquad (30)$$

$$\frac{\partial \mathbf{L}}{\partial L} = \tilde{A}^T Q_\infty + Q_\infty \tilde{A} + \tilde{C}^T \tilde{C} + \gamma^{-2} Q_\infty \tilde{B}\tilde{B}^T Q_\infty = 0 \qquad (31)$$

$$\frac{\partial \mathbf{L}}{\partial G} = 2(\bar{D}_{12}^T \bar{D}_{12} G \bar{C}_2 - \bar{D}_{12}^T \bar{C}_1 - \bar{B}_2 Q_\infty)L\bar{C}_2^T = 0 \qquad (32)$$

Hence, three coupled equations have to be solved simultaneously to obtain the fixed order compensator which satisfies the constraint $\|T_{zw}\|_\infty < \gamma$.

3   Controller Design Using a Homotopy Algorithm

As demonstrated in the previous section, imposing a controller canonical form on the compensator structure provides a powerful tool for the design of fixed order controllers. Promising results have been obtained for the H_∞ and the mixed problem in Ref. [3] where a conjugate gradient method was used. A disadvantage of this method is that convergence slows down near the optimum. Also, an initial starting guess for the compensator gain G has to be provided that stabilizes the closed loop system. In this paper, a homotopy method is employed to perform the fixed order H_∞ controller design. A thorough discussion on homotopy methods and their utilization in fixed order controller design for H_2,

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H_∞ as well as mixed H_2/H_∞ problems is given in Ref. [4]. Therefore, only a concise introduction into the principle of homotopy methods and their implementation in an H_∞ design procedure is provided in this paper.

Homotopy (or continuation) methods, arising from algebraic and differential topology, embed a given problem in a parameterized family of problems. More specifically, consider sets U and $Y \in \Re^n$ and a mapping $F : U \rightarrow Y$, where solutions of the problem

$$F(u) = 0 \tag{33}$$

are desired with $u \in U$ and $F(u) \in Y$. The homotopy function is defined by the mapping $H : U \mathrm{x}[0,1] \rightarrow \Re^n$ such that

$$H(u_1, 1) = F(u) \tag{34}$$

and there exists a known (or easily calculated) solution, u_0, such that

$$H(u_0, 0) = 0 \tag{35}$$

The homotopy function is a continuously differentiable function given by

$$H(u(\alpha), \alpha) = 0, \quad \forall \alpha \in [0,1] \tag{36}$$

Thus the homotopy begins with a simple problem with a known solution (35) which is deformed by continuously varying the parameter until the solution of the original problem (33) is obtained.

For the H_∞ problem two separate homotopy loops are employed. An initial guess for the compensator gain G_0 is obtained by a full order, low authority H_2 design followed by order reduction of the compensator and transformation of the reduced order model into controller canonical form. This procedure was chosen because it has been observed numerically that order reduction techniques tend to work best for low authority LQG controllers [5]. A partial explanation of this phenomenon is given in Ref. [6]. The first

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homotopy transforms the gain G_0 of the low control authority H_2 design to the gain G_1 of a high authority H_2 design. This is done by gradually deforming the weights on control cost and measurement noise while proceeding along the the homotopy path until the desired problem formulation is recovered. A second homotopy is appended to perform the H_∞design. The upper bound for $\|T_{zw}\|_\infty$, γ, is reduced from some high initial value approximating the H_2 design to its minimal value for which a controller exists such that $\|T_{zw}\|_\infty < \gamma$. This procedure is illustrated in Fig. 4. In principle, it is possible to use a direct homotopy from G_0 to G while simultaneously deforming the weights and γ. However, the transition low-high authority always has to be completed in order to obtain the desired plant whereas the minimum value of γ that can be achieved is usually not known beforehand. Thus, the proposed two-step procedure is more feasible for practical purposes.

4   Performance Study for Hypersonic Vehicle

The hypersonic vehicle model used in this study is the Winged-Cone Configuration described in Ref. [7]. In order to carry out control studies, a five state linear model of the longitudinal dynamics is used representing flight conditions for an accelerated flight through Mach 8 at approximately 86000 ft. The model has been normalized in the state variables velocity and altitude to avoid numerical ill-conditioning. State and control variables are:

$$
\mathbf{x} = \begin{bmatrix} V/100 \\ \alpha \\ q \\ \theta \\ h/1000 \end{bmatrix} = \begin{bmatrix} \text{velocity ((ft/sec)/100)} \\ \text{angle of attack (deg)} \\ \text{pitch rate (deg/sec)} \\ \text{pitch attitude (deg)} \\ \text{altitude (ft/1000)} \end{bmatrix} \tag{37}
$$

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$$\mathbf{u} = \begin{bmatrix} \delta e \\ \delta \eta \end{bmatrix} = \begin{bmatrix} \text{symmetric elevon (deg)} \\ \text{fuel equivalence ratio (-)} \end{bmatrix} \tag{38}$$

The interconnection structure for controller design is shown in Fig. 5 and is based on the design in Ref. [1]. Inputs 1 to 5 introduce noise to the measurement signals velocity error V_e, altitude error h_e, normal acceleration n_z, pitch rate q, and pitch attitude θ (output variables 7 to 11). Inputs 6 and 7 are command signals in V and h, inputs 8 and 9 are white noise signals introducing the turbulence components V_g, α_g, and $\dot{\alpha}_g$. Inputs 10 and 11 are the control signals in elevon and fuel equivalence ratio. Outputs 1 to 6 represent the performance variables. A description of the longitudinal model of the vehicle, the Dryden turbulence models and the actuator dynamics is given in Ref. [1]. Also given in Ref. [1] are uncertainty models introducing the effects of the propulsion system, the aeroelastic vehicle behavior and the uncertainty in control effectiveness. These effects, which have a considerable impact on performance, guidance and control characteristics of the vehicle, will be treated in the conference version of this manuscript.

Fig. 6 shows a comparison of the maximum singular value plots of a full order design (12 controller states) vs. 9th, 7th, and 5th order H_∞ controllers. In the current formulation using the controller canonical form the minimal dimension of the compensator is 5 which is determined by the number of measurement outputs. The reduced order controllers come close, but do not fully recover the performance level of the full order controller which was obtained with the two-Riccati-equation solution described in Ref. [8]. This is due to the fact that the closer the homotopy approaches the optimal value of γ, the smaller the stepsize has to be in order to stay on the homotopy path. To reduce computation time, the code was stopped when the stepsize in γ fell short of a prespecified tolerance, in this case $\Delta\gamma_{min} = 0.01$. The values of γ for which the design was stopped (γ_{design}) and the actual values of $\|T_{zw}\|_\infty$ obtained with this design (γ_{actual}) are given in Table 1.

For the 9th order controller, the design value and actual value of γ are very close. This

	full order	9^{th}order	7^{th}order	5^{th}order
γ_{design}	-	0.4796	0.8027	0.7056
γ_{actual}	0.3598	0.4710	0.7390	0.6474

Table 1: γ-values for different controller designs.

indicates that the design is almost optimal although the level of the full order design is not achieved. For the $7^{th}$and 5^{th}order controllers, the gaps between γ_{design} and γ_{actual} are larger implying that these designs can still be improved. In order to do so a finer stepsize would be required to enable the homotopy algorithm to track the solution closer to the optimal γ. An interesting feature is that the 5^{th}order controller achieves a lower γ than the 7^{th}order one. This can be caused by numerical difficulties the homotopy algorithm can encounter along its path forcing the stepsize to be reduced intermediately. This issue will be addressed in future research.

An impression of how well the desired fixed order controllers perform is illustrated in Figs. 7 and 8. They show the velocity and altitude responses for simultaneous step inputs of $V = 100$ ft/sec and $h = 1000$ ft while encountering longitudinal and vertical atmospheric turbulence. It is evident that the responses for all reduced order controllers are almost as good and in most cases even better than the ones of the full order two-Riccati-equation solution. It is interesting to note that in Fig. 7 all systems with reduced order controllers exhibit a non-minimum phase behavior in the velocity response due to the pull-up maneuver to increase the altitude. This behavior may be eliminated by adjusting the weights applied to the error signals V_e and h_e. With the exception of the velocity response of the 5 state compensator all reduced order controllers tend to approach the desired steady-state values at least as good or better than the full order design.

5 Conclusions

A technique for designing fixed order H_∞controllers has been developed and a control study of a hypersonic vehicle has been conducted. A controller canonical form description was imposed on the compensator structure and a homotopy algorithm was used to design H_∞controllers of constrained dimension. A study of achievable nominal performance for various reduced order controllers demonstrates the capability of the developed algorithm and the feasibility of using fixed order control design for hypersonic aircraft. Reducing the allowable minimum stepsize in the homotopy will lead to even better results but will also slow down the procedure. Doing so as well as adjusting the performance weights is expected to reduce or the eliminate non-minimum phase characteristics observed in the velocity response.

Important features typical for hypersonic atmospheric flight like propulsion system interactions and aeroelastic coupling effects are not yet considered. The final manuscript will concentrate on incorporating these features using the uncertainty models of Ref. [1]. Fixed order μ-synthesis design will be used to address these multiple uncertainty sources. Additionally, the issue of further reduction of the controller dimension using an observer canonical form (in place of a controller canonical form) will be investigated, and issues related to controller architecture will be addressed.

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[8] J.C. Doyle et al. "State-Space Solutions to Standard H_2 and H_∞ Control Problems". *IEEE Transactions on Automatic Control*, Vol. 34, No. 8, pp. 832-846, August 1989.
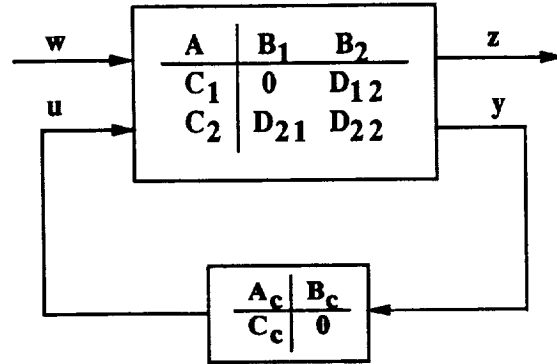
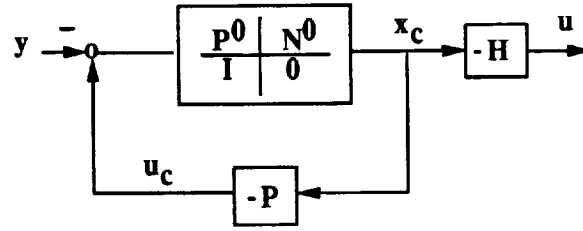Figure 1: Generalized plant with general compensator.

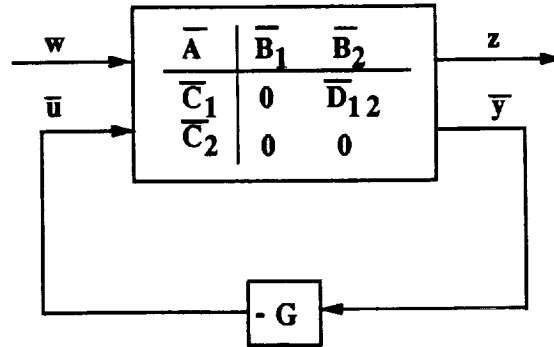
Figure 2: Compensator in controller canonical form.

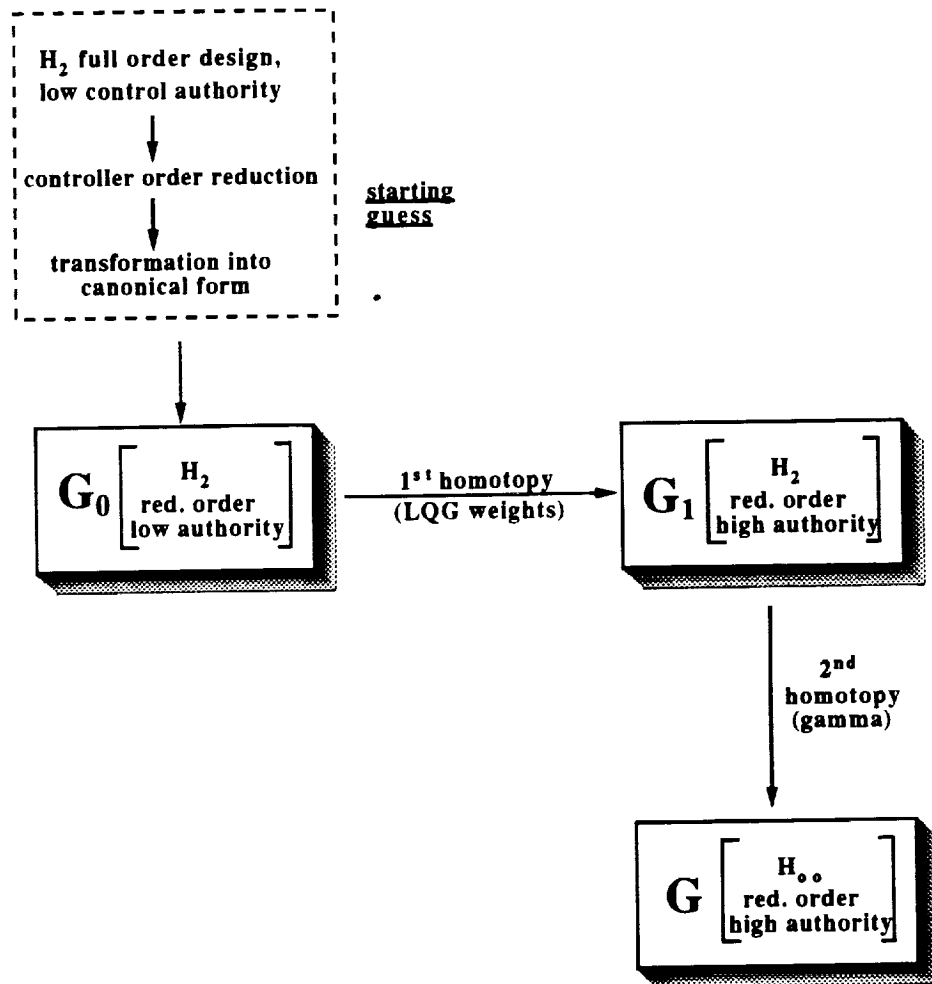Figure 3: Augmented system with compensator in controller canonical form.

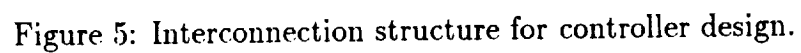Figure 4: Procedure for designing fixed order H_∞ controllers.

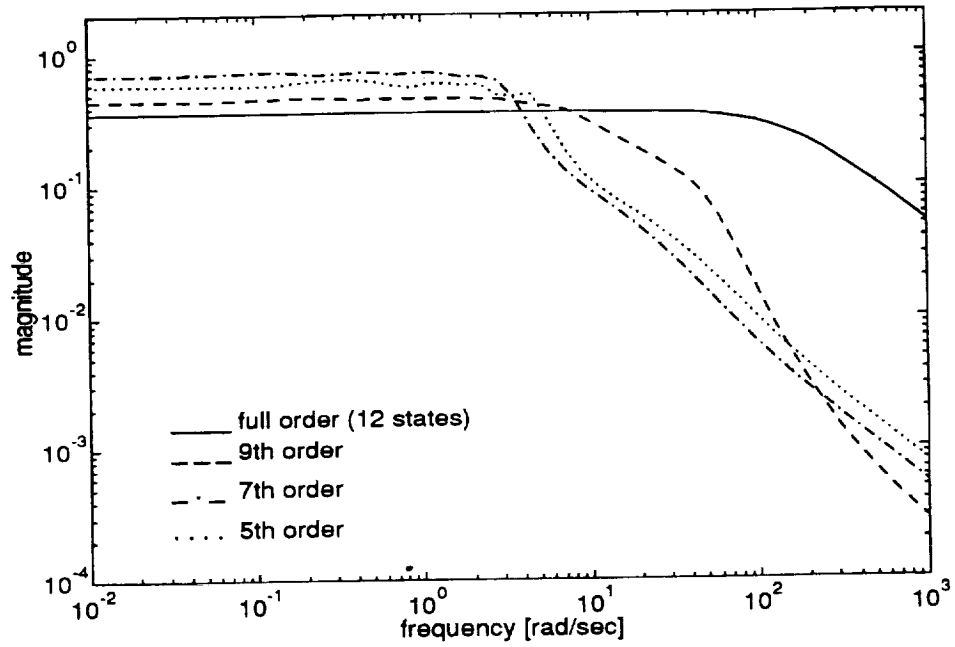Figure 5: Interconnection structure for controller design.

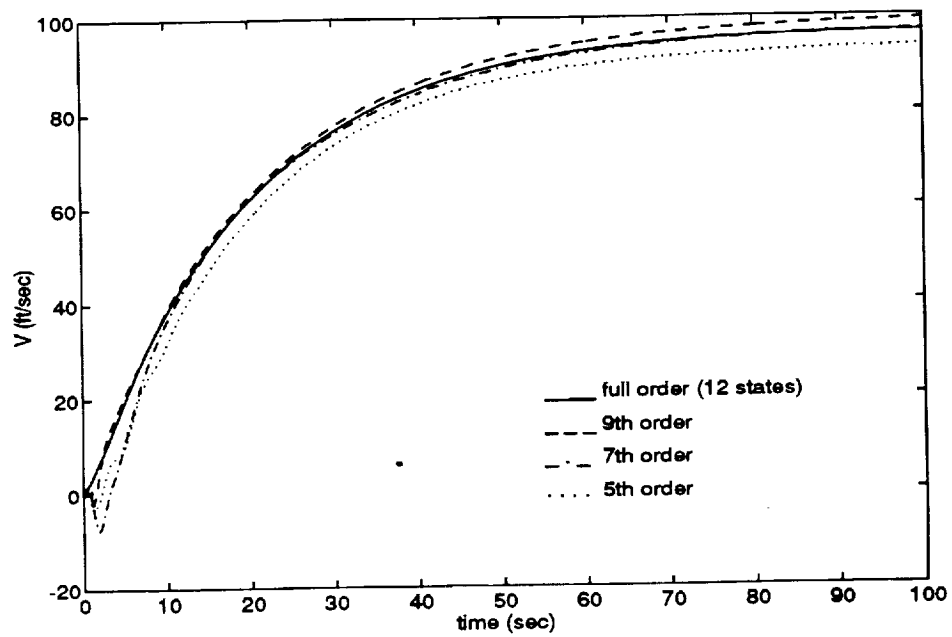Figure 6: Maximum singular value plots for different controller designs.

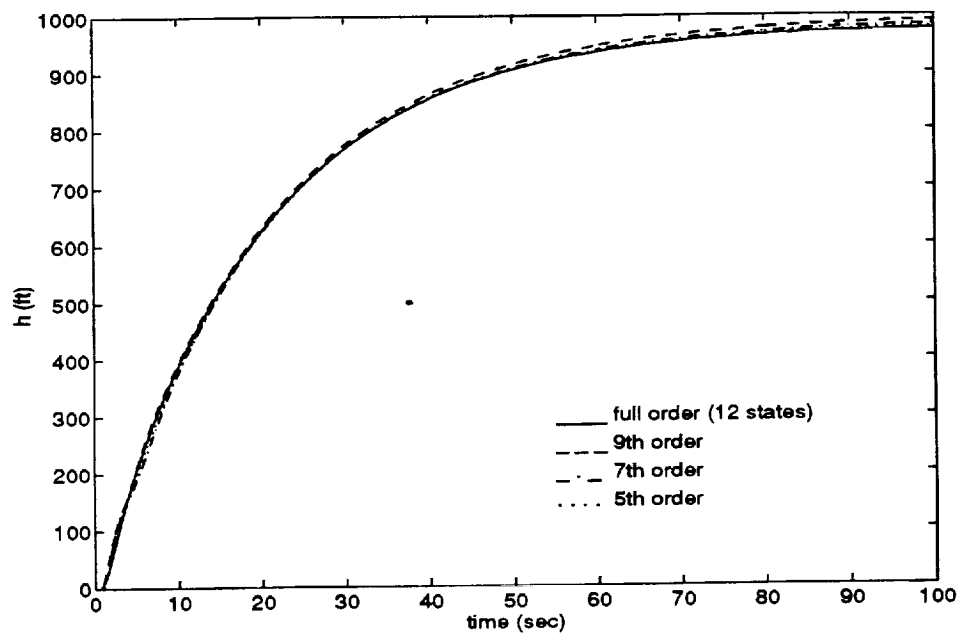Figure 7: Velocity responses for different controller designs.

Figure 8: Altitude responses for different controller designs.